

Transport of heavy metals contaminated leachate through agricultural soil

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ABSTRACT

Contaminations due to heavy metals in soil and groundwater is a widespread problem which affects the fertility and ph level of the soil. Especially considering the agricultural lands that are next to landfills with industrial, medical, domestic and technological waste have a higher risk of getting contaminated. The proposed model deals with two-layer problem containing leachate water from the waste landfill contaminated with heavy metals such as lead, mercury, cadmium, arsenic and chromium. The two-dimensional flow equation for the leachate water section and agricultural soil (red or black soil) with concentration buoyancy effect is studied. With the help of graphs, the results fluid flow, concentration of the heavy metals in the soil are discussed.

1. Introduction:

Agricultural soil such as red or black soil that is near a waste landfill gets contaminated by heavy metals in a course of time. Red soil, which is typically found in tropical and subtropical regions have good drainage and are often acidic in nature. They are rich in iron and aluminum oxides and are often acidic in nature. Black soil also known as Regur soil is high in clay content rich in calcium carbonate, magnesium, potash and lime is known for its moisture retaining capacity.

The leachate water mixed with heavy metals from waste landfills can percolate through soil can transport dissolved heavy metals downward. The red soil which has iron and aluminum oxides can adsorb heavy metals, but cannot retain them because of lower CEC (Cation Exchange Capacity) is much lower than the black soil. The black soil which has high clay and organic matter content can adsorb heavy metals and also has the ability to retain heavy metals. This can lead to accumulation of heavy metals in the soil and become potentially toxic overtime. The soil also loses its fertility and becomes uncultivable terrain.

Bejan [1], explored the role of porous media and its intricate flow structures in various modern engineering applications in the book Porous and Complex Flow structures in Modern Technologies. De Vos [2] investigated the dynamics of water movement and nutrient transport within a stratified silt loam soil. Zhang [3] presented a comprehensive study of stochastic approaches to modeling two-phase fluid flow in porous media. In the book, challenges posed by heterogeneity and uncertainty in porous structures, offering probabilistic and statistical techniques to describe flow and transport phenomena is discussed. Jafari et al., [4] investigated the heat and mass transfer characteristics of viscous fluid undergoing unsteady squeezing flow between two parallel plates. The study examined the influence of key parameters such as the squeeze number, Prandtl number, Eckert number, Schmidt number, and chemical reaction parameter on the flow behavior, temperature distribution, and concentration profiles. Kuechler et al., [5] examined water flow and reactive solute transport in unsaturated subsurface environments. The study focused on

water movement driven solely by rainfall through two distinct soil typer over a typical annual precipitation cycle.

Melnikov et al., [6] investigated the thermodiffusion (Soret effect) in a system comprising a porous layer sandwiched between two fluid layers. The research focused on understanding how the presence of a porous medium influences the separation of components in a binary liquid mixture under thermal gradients. Miyan et al., [7] analyzed the diffusion equations pertinent to fluid movement within porous rocks, focusing the applications of Darcy's law to model such flows. Naeem et al., [8] studied the impact of leachate percolation on heavy metal contamination in soil and groundwater. Nirmala et al., [9] research focused on the dynamics of oil spills on topsoil and the effects of thermal diffusion on oil transport. The paper was developed with a model of two-layer system comprising a horizontal oil layer overlying a subsurface topsoil region saturated with oil and water. Rahman et al., [10] studied the combined influence of thermal and solutal gradients on convective patterns within water-isopropanol mixtures, emphasizing the role of the Soret effect.

Rashid et al., [11] explored the impact of heavy metals and metalloid such as cadmium, lead, arsenic, mercury and chromium on agricultural soils and crop health. The paper highlighted those non-biodegradable pollutants when accumulated to toxic levels, adversely affect crop productivity. Nirmala et al., [12] considered a model which analyzes the heat and mass transfer characteristics of contaminants in groundwater. The research developed a mathematical model comprising partial differential equations to describe the motion of fluid (groundwater) and solid (contaminant) particles. Vafai [13] published a book with an overview of the latest theories and models related to flow, transport, and heat exchange processes in porous media. Vafai [14] offered the fundamental theories and practical applications related to heat and mass transfer in porous materials. Want et al., [15] examined how rainfall influences the release of heavy metals from mining-contaminated soils and the subsequent effects on adjacent farmlands. The research further explored utilizing sequential extraction, batch leaching and dynamic leaching experiments that show that heavy metals such as Cr, Ni, Cu, Zn, As, Cd and Pb were present in concentrations significantly exceeding background levels with predominantly in mobile forms. Zhu et al., [16] employed the D8 single-direction flow algorithm within GIS hydrologic modeling framed to simulate potential flow paths at three specific soil profile. Their findings indicated that the concentrated subsurface lateral flow predominantly appeared at the interfaces involving the clay layer and the underlying bedrock.

This paper deals with contamination of soil from leachate that is mixed with heavy metals. The model developed explores the transport of leachate into agricultural soil and the impact of heavy metals in the soil surface. The effect of the contamination through concentration and temperature profiles is also studied through this model. The objective of this paper is to study the effect of heavy metals contaminated leachate flow into the soil region and observe the concentration and temperature variations inculcated by the leachate.

2. Mathematical Formulation:

The migration of heavy metals filled leachate through agricultural soil is considered. The rectangular coordinate system (x, y) is used to study this model, where x and y represents the x and y axis respectively. The geometry of the model is region with topsoil which is homogeneous, isotropic and the thermo physical properties such as thermal conductivity, viscosity are considered.

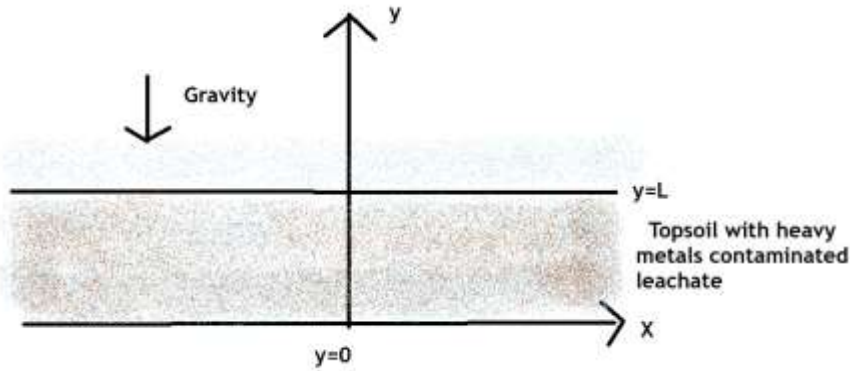


Fig 1: Physical configuration

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g\beta_c(C - C_0) + g\beta_T(T - T_0) - \frac{\sigma\mu\beta_0^2}{\rho} u \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \quad (3)$$

$$= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\phi\beta_c(C - C_0) + g\phi\beta_T(T - T_0) - \phi \frac{v}{k_p} v$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \quad (4)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (5)$$

where u and v are velocities along the x and y directions respectively, t is the time, p represents the pressure on the soil surface, T and C are the temperature and concentration of leachate, T_0 and C_0 are the reference temperature and reference concentration, σ is the presence of electric field, ν is kinematic viscosity of the leachate, ϕ is the porosity, β_c is the concentration buoyancy force, β_T is the temperature buoyancy force, β_0 is the magnetic field strength due to some natural properties of heavy metals, $\phi \frac{v}{k_p} v$ is the Darcy's drag force with permeability term k_p , D is the diffusion coefficient, α is the thermal diffusion term.

Initial and Boundary Conditions:

$$u = 0, v = 0, T = T_0, C = C_0 \text{ at } t = 0$$

$$u = 0, v = 0, T = T_1, \frac{\partial C}{\partial y} = 0 \text{ at } y = 0$$

$$\frac{\partial u}{\partial y} = \frac{a}{\sqrt{k_p}} u, v = v_1, \frac{\partial T}{\partial y} = \frac{a}{\sqrt{k_p}} T, C = C_1 \text{ at } y = L \quad (6)$$

The non-dimensional parameters are as follows

$$x^* = \frac{xU}{\nu}, y^* = \frac{yU}{\nu}, t^* = \frac{tU^2}{\nu}, u^* = \frac{u}{U}, v^* = \frac{v}{U}, p^* = \frac{p}{\rho U^2}, \theta^* = \frac{T - T_0}{T_w - T_0}, \phi^* = \frac{C - C_0}{C_w - C_0}$$

Applying the non-dimensional parameters to the equations (1) to (5),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + Gr_T \theta + Gr_C \phi - Ha^2 u \quad (8)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \varphi Gr_T \theta + \varphi Gr_C \phi - \sigma^2 \varphi v \quad (9)$$

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) \quad (10)$$

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (11)$$

where $Ha^2 = \frac{\sigma v^2 \beta_0^2}{U^3}$ is the Hartmann number, $Sc = \frac{\nu}{D}$ Schimdt number and $Pr = \frac{\nu}{\alpha}$ Prandtl number, Gr_C and Gr_T is the mass Grashof number and thermal Grashof number respectively.

The initial and boundary conditions after applying non-dimensional parameters are as follows,

$$u = 0, v = 0, \phi = 1 \text{ for } t = 0$$

$$u = 0, v = 0, \theta = T_x, \frac{\partial \phi}{\partial y} = 0 \text{ at } y = 0 \quad (12)$$

$$\frac{\partial u}{\partial y} = a\sigma u, v = v_1, \frac{\partial \theta}{\partial y} = a\sigma(\theta + T_x) \text{ at } y = 1$$

3. Method of Solution:

The solution for the velocity, concentration and temperature are obtained used Perturbation method. Since the equations from (7) to (11) are two-dimensional and nonlinear, the solutions are expressed through the perturbation method assuming perturbation parameter ϵ to be small.

$$\begin{aligned} u(x, y, t) &= u_0(y) + \epsilon e^{(\lambda x + \omega t)} u_1(y) + o(\epsilon^2), \\ v(x, y, t) &= \epsilon e^{(\lambda x + \omega t)} u_1(y) + o(\epsilon^2), \\ p(x, y, t) &= p_0(y) + \epsilon e^{(\lambda x + \omega t)} u_1(y) + o(\epsilon^2) \\ \phi(x, y, t) &= \phi_0(y) + \epsilon e^{(\lambda x + \omega t)} \phi_1(y) + o(\epsilon^2) \\ \theta(x, y, t) &= \theta_0(y) + \epsilon e^{(\lambda x + \omega t)} \theta_1(y) + o(\epsilon^2) \end{aligned} \quad (13)$$

Zeroth order equations:

By substituting equation (13) in equation (7) through (11) and neglecting the higher order ϵ^2 , the following zeroth order equations for velocity, temperature and concentration are obtained. The initial and boundary conditions in equation (12) are also modified to satisfy the zeroth order equation.

Velocity:

$$0 = -\frac{\partial p_0}{\partial x_0} + \frac{\partial^2 u_0}{\partial y^2} + Gr_T \theta_0 + Gr_C \phi_0 - Ha^2 u_0 \quad (14)$$

The initial and boundary conditions of velocity are

$$u_0 = 0 \text{ at } y = 0 \quad (15)$$

$$\frac{\partial u_0}{\partial y} = a\sigma u_0 \text{ at } y = 1$$

$$u_0 = \frac{r_4 [e^{\sqrt{N}y} - 1] - \sinh \sqrt{N}y [r_5 (\frac{r_6}{\sqrt{N}} - e^{\sqrt{N}})]}{r_7} \quad (16)$$

where $r_1 = Ha^2, r_2 = Gr_T \theta_0, r_3 = Gr_C \phi_0, r_4 = \frac{r_1 - r_2 - r_3}{N}, r_5 = \frac{r_1 - r_2 - r_3}{\sqrt{N}}, r_6 = a\sigma(e^{\sqrt{N}y} - 1),$
 $r_7 = 2\sin \sqrt{N}y - \sqrt{N} \cosh \sqrt{N}$

Temperature:

$$\frac{\partial^2 \theta_0}{\partial y^2} = 0 \quad (17)$$

subject to the initial and boundary conditions

$$\theta_0 = T_{x_2} \text{ at } y = 0 \text{ and } \frac{\partial \theta_0}{\partial y} = a\sigma(\theta_0 + T_{x_3}) \text{ at } y = 1 \quad (18)$$

$$\theta_0 = r_8 y + T_{x_2} \quad (19)$$

where $r_8 = \frac{a\sigma(\theta_0 + T_{x_3})}{1 - a\sigma}$

Concentration:

$$\frac{\partial^2 \phi_0}{\partial y^2} = 0 \quad (20)$$

The initial and boundary conditions for the temperature equation is

$$\frac{\partial \phi_0}{\partial y} = 0 \text{ at } y = 0 \text{ and } \phi_0 = C_{x_1} \text{ at } y = 1 \quad (21)$$

$$\phi_0 = C_{x_1} \quad (22)$$

1st order Perturbation equation:

The following equation is the first order perturbation equation for velocities u and v.

$$\left(\frac{d^2 u_1}{dy^2} - \lambda^2 u_1 \right) - (\omega + \lambda u_0) u_1 + u'_0 v_1 = -\frac{\lambda}{\rho} p_1 + Gr_T \theta_1 + Gr_C \phi_1 - Ha^2 u_1 \quad (23)$$

where $u'_0 = \frac{du_0}{dy}$

$$\frac{1}{\phi} \frac{d^2 v_1}{dy^2} - \left(\frac{\omega}{\phi} - \frac{\lambda^2}{\phi} + \frac{u_0 \lambda}{\phi} + \frac{1}{\sigma^2} \right) v_1 = -p_1 \quad (24)$$

4. Perturbed Part:

Introducing the stream functions $u_1 = -\frac{\partial \psi_1}{\partial y}$, $v_1 = \lambda \psi_1$, which is an useful mathematical method that can be used to solve the continuity and momentum equations directly for a single variable.

By eliminating the pressure term from equations (23) and (24),

$$\psi_1'''' - (\lambda^2 + \omega + \lambda u_0 + Ha^2) \psi_1'' - \lambda u'_0 \psi_1 - \frac{1}{\phi} \lambda \psi_1'' + \left(\frac{\omega}{\phi} - \frac{\lambda^2}{\phi} + \frac{u_0 \lambda}{\phi} + \frac{1}{\sigma^2} \right) \lambda \psi_1 = Gr_T \theta_1' + Gr_C \phi_1' \quad (25)$$

$$\frac{\phi_1''}{Sc} - \lambda \psi_1 \phi_0' + \left(\frac{\lambda^2}{Sc} - \omega + u_0 \lambda \right) \phi_1 = 0 \quad (26)$$

$$\frac{\theta_1''}{Pr} - \lambda \psi_1 \theta_0' + \left(\frac{\lambda^2}{Pr} - \omega + u_0 \lambda \right) \theta_1 = 0 \quad (27)$$

The modified initial and boundary conditions in terms of ψ_1 are

$$\begin{aligned} u &= 0, v = 0, \phi = 1 \text{ at } t = 0 \\ \psi_1 &= 0, \psi_1' = 0, \theta_1 = T_x, \phi_1' = 0 \text{ at } y = 0 \\ \psi_1 &= \frac{v_1}{\lambda}, \psi_1'' = a\sigma \psi_1', \theta_1' = a\sigma(\theta_1 + T_x) \text{ at } y = 1 \end{aligned} \quad (28)$$

where the prime (') denotes the differentiation with respect to y.

The equations from (25) to (27) subject to the initial and boundary conditions in equation (28) are solved numerically using Mathematica 13.0

Differentiating the solutions of ψ_1 with respect to y gives the perturbed axial velocity u. The zeroth order solutions given equation (16), (19) and (22) along with the obtained numerical solutions gives the velocity, temperature and concentration distribution of heavy metals contaminated leachate into the soil region.

5. Results and Discussion:

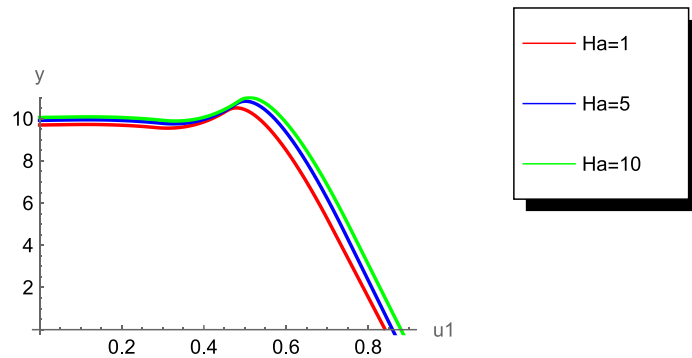


Fig 2: Axial velocity with the variation of Hartmann number.

Figure 2 shows the axial velocity with varying Hartmann number from 1 to 10. The figure illustrates that an increase in Hartmann number reduces axial velocity due to the amplified magnetic damping effect. All three curves display a peak in axial velocity before decreasing sharply. Due to a balance between inertial forces and the opposing magnetic force, the peak remains in the same location but becomes slightly more noticeable at higher Ha . It is observed that at the boundary, the velocity increases with the increase in Ha , which indicates a stronger damping effect near the walls. This may be due to the increase in Ha thickens the boundary layer leading to increased resistance against fluid motion.

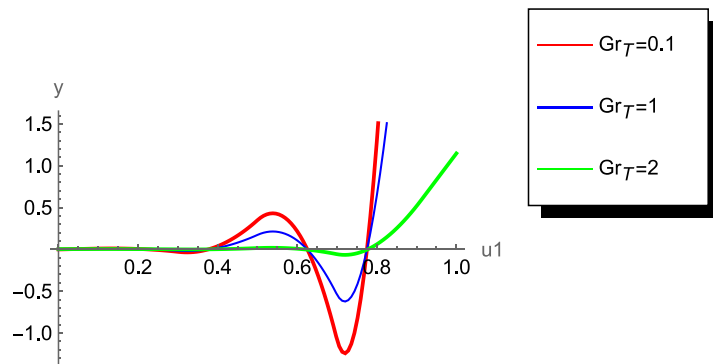


Fig 3: Effects of thermal Grashof number on axial velocity.

Effects of thermal Grashof number on axial velocity is observed in Figure 3. Higher thermal Grashof number indicates an increase in buoyancy-driven forces, that enhances fluid acceleration. When the buoyancy force term exceeds a certain point, it overcomes viscous damping force, leading to a reversal in the axial velocity (u_1 changes sign). The presence of the heated surface implies thermal boundary layers (in agricultural soil), where buoyancy-driven recirculation can arise. In the lower boundary region, buoyancy force pulls fluid which are less viscous upwards, but the inertia of the surrounding fluid will resist this motion. The leachate migrating into the soil will face resistance due to the buoyancy force which leads to velocity of the fluid slowing down and forming a reversal effect which is shown in the graph as concave down (dip in the curve) of the curve.

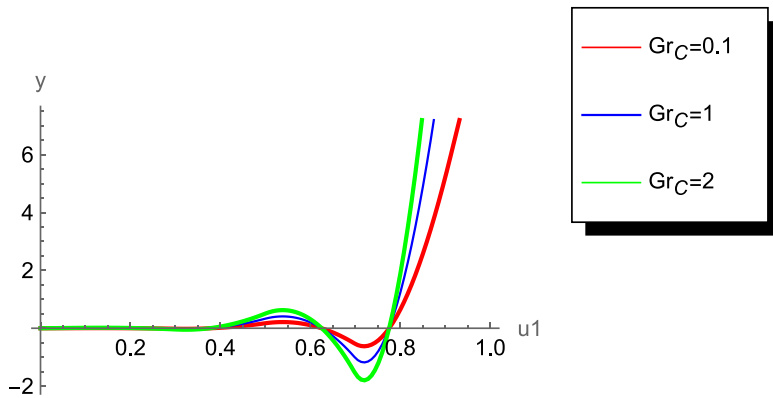


Fig 4: Mass Grashof number's effects on axial velocity.

Axial velocity reaction with the change in mass Grashof number perceived in Figure 4. The low Gr_C leads to weaker buoyancy force which makes the velocity profile to take a slight dip. This shows that a minor opposing flow region before the dominant flow takes over. The dip in the velocity can occur due to the contaminant's interaction with soil particles. As the resistance increases causing a temporary slowdown in migration. The dip in the curves represent a phase where contaminants (heavier metals) settles and adsorption to soil particles slows down migration. When Gr_C becomes higher, the contaminants migrate faster into the soil leading to increase in the risk of groundwater contamination.

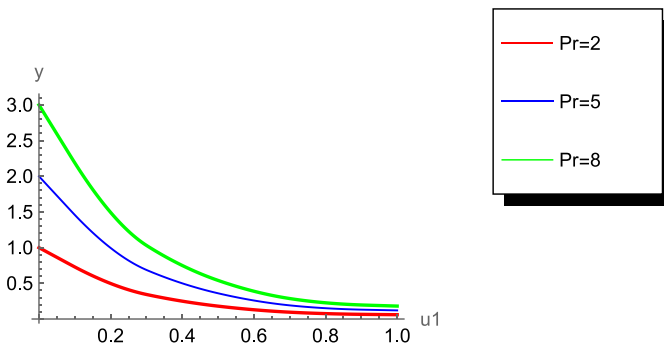


Fig 5: Temperature distribution due to Prandtl number.

Figure 5 depicts the effects of Prandtl on temperature distribution. The graph shows the temperature curves for the values of $Pr = 2, 5$ and 8 . Understanding the thermal behavior of the contaminants is significant for predicting and mitigating heavy metal migration in soil region. The increase in Pr leads to sharper decay in temperature, implying stronger thermal boundary effects. The decrease in Pr allows heat to diffuse more widely affecting contaminant transport. Higher Pr values does not allow heat from leachate to spread quickly keeping the contaminants in localized zones. Slower heat distribution leads to longer retention of pollutants in upper soil region. Lower Pr values indicate the heat spreads as quickly as possible accelerating chemical reactions induced by heavy metal mobilization.

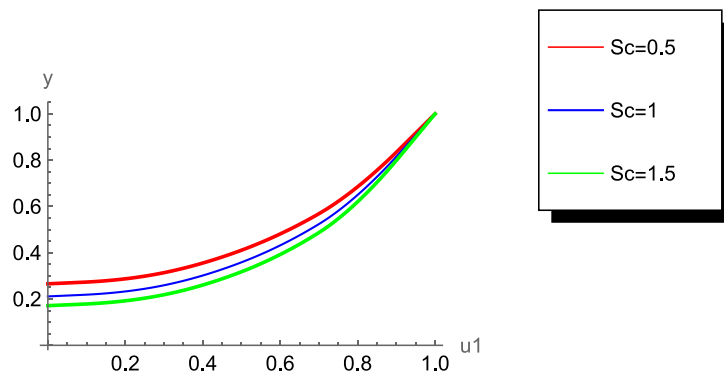


Fig 6: Concentration profile under the influence of Schmidt number.

Schmidt number effects on concentration profile is studied in figure 6. The graph represented is varied for $Sc = 0.5$ to 1.5 . The contaminants spread more easily for lower Sc . The steeper curve ($Sc = 1.5$) indicates slower contaminant migration due to the higher value of Schmidt number. The lower Sc value helps the heavy metals or chemical leachates infiltrate deeper into agricultural soil and contaminate the groundwater faster. With high Sc values, contaminants diffuse slowly and remain concentrated on the surface of the soil. Heavy metals such as lead, cadmium and mercury tend to have higher Schmidt numbers, which makes them accumulate on the soil surface instead of spreading quickly. This leads to soil infertility which affects crop growth on the agricultural soil.

7. Conclusion:

The paper investigated the transport of heavy metals contaminated leachate from landfill through agricultural soil, concentrating on the dispersion and adsorption mechanism that influences heavy metal migration and retention. The transport of contaminants is significantly affected by the soil composition, porosity and moisture content. Perturbation technique is used to find the velocity, temperature and concentration of leachate in soil. By using the Hartmann number, thermal Grashof number, mass Grashof number, Schmidt number and Prandtl number, the contaminant migration is studied. The graphs show the influence of non-dimensional numbers that plays an important role in migration of the leachate into the soil profile. The model developed also discuss the temperature and concentration of the leachate affecting the fertility of the soil. The model can be further improved by adding soil parameters such as organic content and microbial activity to refine the predictions. The paper can also be expanded for multiphase flow analysis by varying hydraulic gradients in contaminant transport.

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