Radio Analytic Mean Graceful Number of Star and Bistar Related Graphs

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KEYWORDS

Radio analytic mean labelling , graceful graph, star graph , Bistar graphs , diameter, Labelling

ABSTRACT

In this paper, we introduce the concept of star and Bistar-related graceful graphs that satisfy the radio analytic mean condition. Specifically, a radio analytic mean labeling of a graph G=(V,E) is defined
as a function f:V(G)→N f: V(G) }such that for any two vertices u and v, the following condition holds
d(u,v) + ⌈| 〖θ(u)〗 ^2- 〖θ(v)〗 ^2 |/2⌉ ≥ 1+ diam (G) . where d(u,v)d(u, v)d(u,v) represents the distance between any two vertices u and v in G, and diam(G) is the diameter of the graph G.The radio analytic mean number ramgn(f) is the maximum integer allocated to any node of G, and this node is referred to as the radio analytic mean graceful number.In this article, we investigate the ramgn of certain star and Bistar-related graphs, exploring how these graphs satisfy the radio analytic mean condition and examining their properties.Where d(u,v) is distance between any two nodes of G. A radio analytic mean number ramgn (f) is maximum integer allocated to any node of G. In this node is called radio analytic mean graceful number. In this article we investigate ramgn certain star and Bistar Related graphs.

I : INTRODUCTION

Graph theory plays an essential role in various fields of mathematics, including communication systems, robotics, neural networks, physical sciences, MATLAB theory, and signal systems. Researchers have developed numerous types of graph labeling, such as radio labelling, radio mean labelling, and radio analytic mean labelling, radio Geometry to address different mathematical challenges. Graph labelling involves assigning labels to the edges and nodes of a graph while adhering to specific conditions. One of the early and important concepts in graph labelling was graceful labeling, introduced by Rosa in 1967. Graceful labelling is defined as a function f that maps the vertices of a graph G to the set {0,1,2,3,…,q } such that for every edge uv=e, the label assigned to the edge is |f(u)−f(v)||. The crucial condition in graceful labelling is that all the edge labels must be distinct.Later, in 2001, Chartrand introduced the concept of radio labelling, which extends the ideas of graceful labelling. This concept involves assigning labels to the vertices of a graph while considering additional conditions related to the distance between vertices.

In 2015, Ponraj introduced the concept *of* radio mean labeling and the related concept of the cycle-related radio mean subdivision number. These concepts further expanded the applications of radio labeling by considering additional structural properties of graphs.
In 2019, P. Poomalai defined the concept of radio analytic mean labeling and formulated the radio analytic mean D-distance *in* various types of graphs. This work deepened the understanding of how graph labelling can be

used to analyse in graceful graph the relationships and distances between vertices in a more complex and analytical manner

II: PRELIMINARIES

Definition 2.1: The graph S ($k_{1,n}$) is a star graph , n ≥3

Definition 2.2: Let w be the centre vertex and $v_0,v_1,v_2......v_n$ be the pendent vertices are called fan graph

Definition 2.3: The Bistar $B_{x,y}$ is the graph received by joining the two central vertices of ($k_{1,n}$)

Definition 2.4: The graceful labeling of a graph G=(V,E), where G has n vertices and q edges, is defined by a bijective function f that maps the set of vertices Vto the set {0,1,2,…,q}. In this labelling, for every edge xy∈E, the label assigned to the edge is given by ∣f(x)−f(y)∣. The key condition for graceful labelling is that all edge labels must be distinct, meaning that no two edges share the same label..

Definition 2.5 : The diameter of a graph G is defined as the length of the longest shortest path between any two nodes in the graph. In other words, it is the greatest distance (in terms of the number of edges) you must travel between any two vertices to reach one another by the shortest path.

Definition 2.6: The d(u,v) is the distance between arbitrary two neighbour vertex of G.

III: Results and Discussion:

Theorem 3.1 : The Radio Analytic mean Graceful number of star graph , S ($k_{1,n}$) = n , n ≥3

Proof: Consider u be the centre vertex of star graph. Let $u_1,u_2......u_n$ be the pendent vertices of S ($k_{1,n}$) . The diameter of S ($k_{1,n}$) is 2. By def (2.4) edge labels are distinct. The star graceful graph satisfied radio analytic mean condition.

$d(u,v) + \left\lceil\frac{|\theta(u)^2-\theta(v)^2|}{2}\right\rceil \geq 1+ \text{ diam } (G)$ …………………… (1)

We define the function f: S ($k_{1,n}$) → z , f (u) = n , $f(u_i)= i$, $0\leq i\leq n$

We realize following possible cases satisfied the Equation (1)

(i)View the pair $d(u,u_i)$, where $0\leq i\leq n$, $d(u,u_i) + \left\lceil\frac{|f(u)^2-f(v)^2|}{2}\right\rceil \geq 3$

$$=> 1 + \left\lceil\frac{|(n)^2-(i)^2|}{2}\right\rceil \geq 3$$

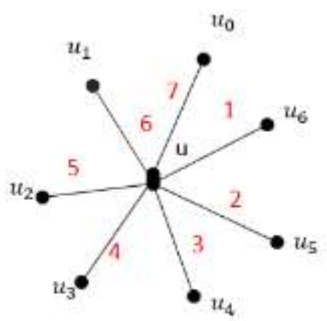
(ii) View the pair $d(u_i,u_j)$, where $0\leq i\leq n$, $0\leq j\leq n$, $d(u_i,u_j) + \left\lceil\frac{|(i)^2-(j)^2|}{2}\right\rceil \geq 3$

$$=> 2 + \left\lceil\frac{|(4)^2-(6)^2|}{2}\right\rceil \geq 3$$

$$=> 12 \geq 3$$

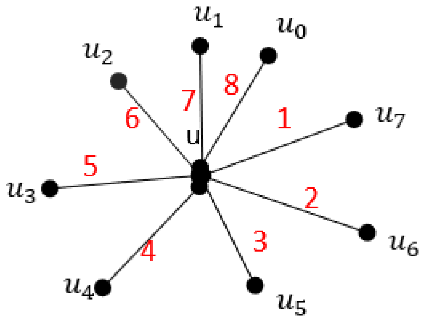
Therefore radio analytic mean graceful number of S ($k_{1,7}$) = 7, n ≥3.

Illustration 1: case n is odd



ie) Ramgn S ($k_{1,7}$) = 7

Illustration 2: case n is even



ie) Ramgn S (k $_{1,8}$) = 8

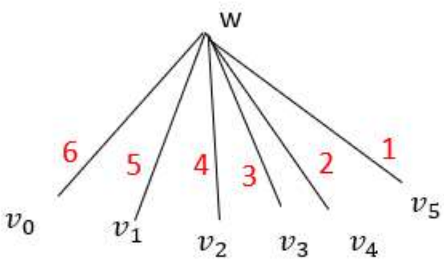
Theorem 3. 2: The Radio Analytic mean graceful number of fan graph Fn = n, n≥ 6
Proof :

A fan graph constructed by joining all pendent vertices of $F_{n,}$ =n, n≥ 6 is a path graph on n nodes. Here diameter of fan graph is 2. We define the function $(F_n) \rightarrow$ z by , f (w) = n , f (v_i) =i, 0≤ i≤ n -1 .

$$d(u,v) + \left\lceil \frac{|\theta(u)^2 - \theta(v)^2|}{2} \right\rceil \geq 1 + \text{diam}(G) \ldots\ldots\ldots\ldots \quad (2)$$

Here all the arbitrary pairs satisfied inequality (2) .

Illustration 3:



ie) ramg F_6 = 6

Theorem 3.3: The Radio analytic mean graceful number of complete Bipartite graph Km,l = 3m
Proof : we define the function f (x_i) = i, 0≤ i≤ $m-1$, f(y_j)= 2n , 4≤ j≤ $m+2$, if n= m-2, m, m+2.
Let f: V(G) = $\{0,1,2 \ldots\ldots q\}$ then f: E(G) = $\{1,2 \ldots\ldots q\}$ edge labels are distinct. This resultant graph is called graceful graph. This graceful graph satisfied inequality d(u,v) + $\left\lceil \frac{|\theta(u)^2-\theta(v)^2|}{2} \right\rceil$ ≥ 1+ diam (G) …… (3).

Here diam $K_{m,l}$ =2 . (i)View the pair d(x_i ,y_j), , 0≤ i≤ $m-1$, 4≤ j≤ $m+2$, d(x_i ,y_j) + $\left\lceil \frac{|(i)^2-(2n)^2|}{2} \right\rceil$ ≥ 3

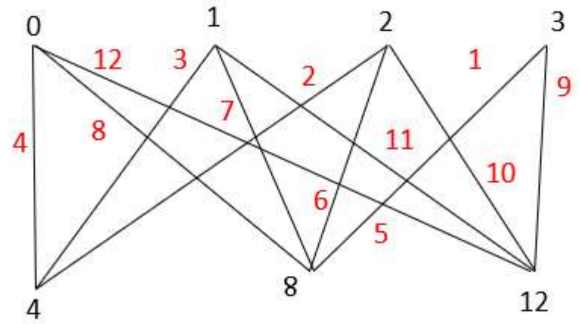
(ii) View the pair d(x_i ,x_j), , 0≤ i≤ j ≤ $m-1$, d(x_i ,x_j) + $\left\lceil \frac{|(i)^2-(j)^2|}{2} \right\rceil$ ≥ 3

(iii) View the pair d(y_m ,y_{m+1}), if d(y_m ,y_{m+1})+ $\left\lceil \frac{|(4)^2-(8)^2|}{2} \right\rceil$ ≥ 3

(iv) View the pair d(y_{m+1} ,y_{m+2}), if d(y_{m+1} ,y_{m+2})+ $\left\lceil \frac{|(8)^2-(12)^2|}{2} \right\rceil$ ≥ 3

Clearly f is an bijective function and we shows that complete bipartite graceful graph f satisfied radio analytic mean condition d(u,v) + $\left\lceil \frac{|\theta(u)^2-\theta(v)^2|}{2} \right\rceil \geq 1+$ diam (G). Therefore the greatest integer allocated to any arbitrary vertex below this labelling is 3n.

Illustration 4:



ie) $\mathbf{K_{4,3} = 12}$

Theorem 3.4 : The Radio Analytic mean graceful number of S(k $_{1,n}$) is 3n , n ≥3

Proof: Consider $u_1, u_2, u_3 \ldots\ldots u_n$ be the pendent vertices of u and u be the apex vertex of S(k $_{1,n}$) and Let $v_1, v_2 \ldots\ldots v_n$ be the connected vertices of u and v . The Non negative greatest integer assign to any vertex of S(k $_{1,n}$) . Let G be a graph with p vertices and q edges. we define vertex labelling f: V(G) = $\{0,1,2 \ldots\ldots q\}$ then f: E(G) = $\{1,2 \ldots\ldots q\}$ edge labels are distinct. This resultant graph is called graceful graph. This graceful graph satisfied the radio analytic mean condition equation as follows

d(u,v) + $\left\lceil \frac{|\theta(u)^2-\theta(v)^2|}{2} \right\rceil \geq 1+$ diam (G) …………………… (4)

Clearly f is hold one to one and onto function , then it is called bijective function . The S(k $_{1,n}$) is diameter is 3. We define a function f: V(S (k $_{1,n}$) → z^+

f(u) = n , f(u_i) = n+i ,1≤ i≤ n , f(v) =0 , f(v_i) = 2n +i , 1≤ i≤ n and f(e_i) =i , 1≤ i≤ q .Here order of the vertices v(G) = 2n+2 and order of the edges E(G) = 3n .

Now we have to prove Ramc Equality condition (4) as follows

(i) View the pair d(u ,u_i), i≥1, d(u, u_i) + $\left\lceil \frac{|(n)^2-(n+i)^2|}{2} \right\rceil \geq 4$

(ii) View the pair d(u_i ,u_j), i≥ j ≥1, d(u_i, u_j) + $\left\lceil \frac{|(n+i)^2-(n+j)^2|}{2} \right\rceil \geq 4$

(iii) View the pair d(u ,v_i), i≥1, d(u, v_i) + $\left\lceil \frac{|(n)^2-(2n+i)^2|}{2} \right\rceil \geq 4$

(iv) View the pair d(v_i ,v_j), i≥ j ≥1, d(v_i, v_j) + $\left\lceil \frac{|(2n+i)^2-(2n+j)^2|}{2} \right\rceil \geq 4$

(v) View the pair d(u_i,v_i) , i≥1, d(u_i,v_i)+ $\left\lceil \frac{|(n+i)^2-(2n+i)^2|}{2} \right\rceil \geq 4$

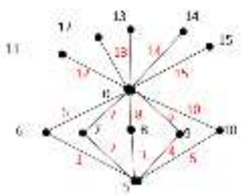
(vi) View the pair d(u,v) , d(u,v) + $\left\lceil \frac{|(n)^2-(0)^2|}{2} \right\rceil \geq 4$

It is obvious from above labelling integers that f have inequality (4) .

Therefore all the cases, the inequality hold and the S(k $_{1,n}$) graceful graph satisfies the radio analytic mean condition . The highest integer 3n is assigned to any vertex v_i .

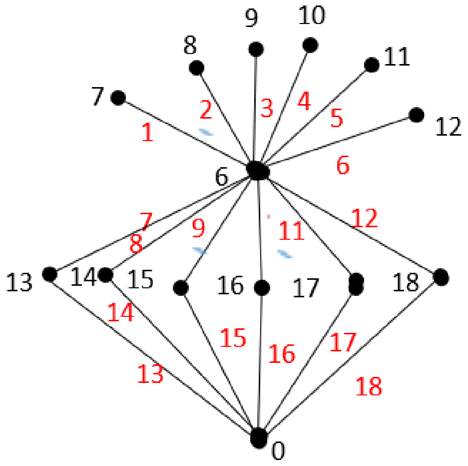
Finally The S (k $_{1,n}$) shows to Radio analytic mean graceful number 3n , n ≥3.

Illustration 5:



ie) Ramg S(k $_{1,5}$) =15

Illustration 6:



ie) Ramg S($k_{1,6}$) =18

Theorem 3.5: The Radio Analytic mean graceful number of $B_{n,n}$ is 6n+3, n≥3

Proof: f(u)=4n+2,

f($u_i{}'$) =2n+i, 1≤ i≤ n-1, f($u_i{}'$) =4n, i=n=5, f(v)=4t+3 if t=n, f($v_i{}'$) =2n-2i, 0≤i≤n-1 ,f(w)=0,

f($w_i{}^{i}$) = 5n+3+i , 1≤ i≤ n, f(s)= 4n+1 , f($s_i{}'$)= 4n+3+i , 1≤ i≤ n

Define a function f: V(Bn,n) is non negative integers. Consider the vertex set Bn,n is $\{u, u_i{}' , v, v_i{}' , w, w_i{}', S, S_i{}'\}$ 1≤ i≤ n .Here f ($u_i{}'$) to be pendent vertices of u. f($v_i{}'$) is corresponding to pendent vertices of u . it is obivisiouly $w_i{}', S_i{}'$are the pendent vertices of f(w) and f(s).its diameter is 3. Clearly f is bijective function. we allocate the labelling f: V(G)= $\{0,1,2 \ldots .p\}$and f: E(G)= $\{1,2 \ldots .q\}$ .In this graph is called graceful graph . The graceful graph satisfied the Ramc.

ie) d(u,v) + $\left\lceil \frac{|\theta(u)^2-\theta(v)^2|}{2} \right\rceil$ ≥ 1+ diam (G) ………… (5)

case (i) d(u,v)= k if k= 2

(1)View the pair d($u_i{}'u_j{}'$) ≥ 2 , i ≠j , d($u_i{}'u_j{}'$) + $\left\lceil \frac{|(2n+i)^2-(2n+j)^2|}{2} \right\rceil$ ≥ 4

(2)View the pair d($v_i{}'v_j{}'$) ≥ 2 , i ≠j , d($u_i{}'u_j{}'$) + $\left\lceil \frac{|(2n-2i)^2-(2n-2j)^2|}{2} \right\rceil$ ≥ 4

(3)View the pair d(u,v) ≥ 2 , d(u,v) + $\left\lceil \frac{|(4n+2)^2-(4n+3)^2|}{2} \right\rceil$ ≥ 4

(4) View the pair d($w_i{}'w_j{}'$) ≥ 2 , i ≠j , d($w_i{}'w_j{}'$) + $\left\lceil \frac{|(5n+3+i)^2-(5n+3+j)^2|}{2} \right\rceil$ ≥ 4

(5)View the pair d($s_i{}' s_j{}'$) ≥ 2 , i ≠j , d($s_i{}'s_j{}'$) + $\left\lceil \frac{|(4n+3+i)^2-(4n+3+j)^2|}{2} \right\rceil$ ≥ 4

(6)View the pair d($w_i{}', s_i{}'$) ≥ 2 , d($w_i{}'s_i{}'$) + $\left\lceil \frac{|(5n+3+i)^2-(4n+3+i)^2|}{2} \right\rceil$ ≥ 4

(7)View the pair d(w,s) ≥ 2 , d(w,s) + $\left\lceil \frac{|(0)^2-(4n+1)^2|}{2} \right\rceil$ ≥ 4

Here all the pairs satisfied the inequalities (5) and this graceful graph Called Ramg.

Case (ii) : d(u,v) =k ,if k = 3

(8)View the pair d($u_i{}' w_i{}'$) ≥ 3 , Where 1≤ i≤ n , d($u_i{}'w_i{}'$) + $\left\lceil \frac{|(2n+i)^2-(5n+3+i)^2|}{2} \right\rceil$ ≥ 4

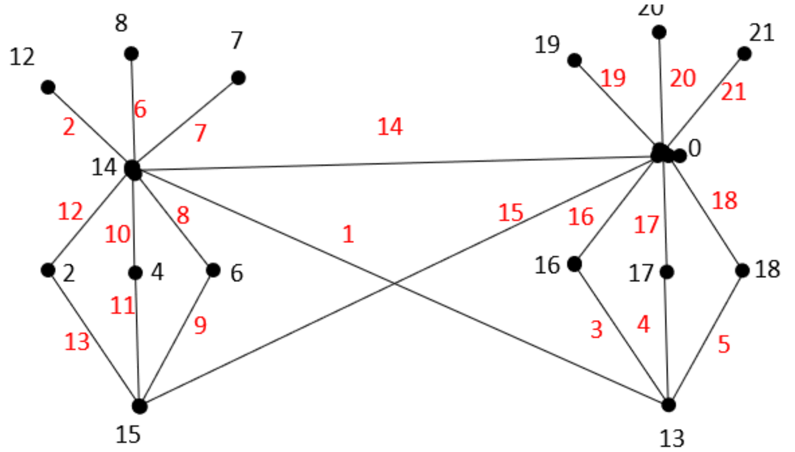
(9) View the pair d($v_i{}', w_i{}'$) ≥ 3 , Where 1≤ i≤ n , d($v_i{}'w_i{}'$)+ $\left\lceil \frac{|(2n-2i)^2-(5n+3+i)^2|}{2} \right\rceil$ ≥ 4

(10) View the pair d($v_i{}', s_i{}'$) ≥ 3 , Where 1≤ i≤ n , d($v_i{}'w_i{}'$)+ $\left\lceil \frac{|(2n-2i)^2-(4n+3+i)^2|}{2} \right\rceil$ ≥ 4

(11) View the pair d(v,s) ≥ 3 , d(v,s) + $\left\lceil \frac{|(4n+3)^2-(4n+1)^2|}{2} \right\rceil$ ≥ 4

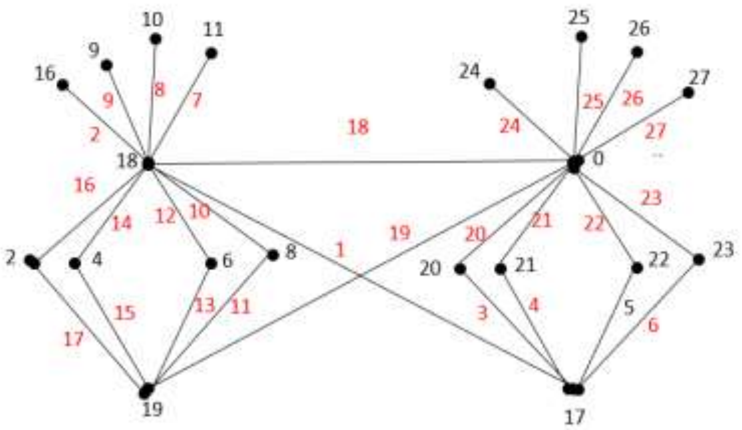
It is obvious from above labelling integers that f have inequality (1) .
Therefore all the cases ,the inequality hold and the $B_{n,n}$ is 6n+3, n≥3
$B_{n,n}$ is every pair of graceful graph satisfies the radio analytic mean condition . The highest integer 6n+3 is assigned to any vertex u_i'. Finally $B_{n,n}$ shows to Radio analytic mean graceful number is 6n+3, n≥3

Illustration 7:



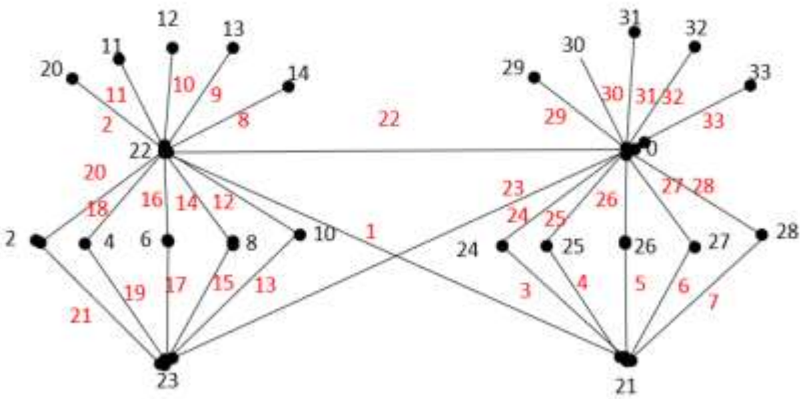
ie) ramg ($B_{3,3}$) = 21

Illustration 8:



ie) ramg ($B_{3,3}$) = 21

Illustration 9 :



ie) ramgn ($B_{5,5}$) =33

Theorem 3.6: The Radio Analytic mean number of graceful graph $B^2(n,n)$ is 4n+1, n≥ 7

Proof:

We assign the vertex set Bn,n is $\{u, w, u_i', w_i', 1 \le i \le n\}$. Here u and w are upper and lower vertex of B^2(n,n). The diameter of B^2(n,n) is 2. Where u_i', w_i' are the pendent vertices of u and w. Let the graph B^2(n,n) then V(G) = $\{0,1,2, \dots \dots 2n + 2\}$vertices and edge set E(G) = $\{1,2, \dots \dots 2n + 2\}$ and edge labels are distinct . Therefore B^2(n,n) graph is called graceful graph. Let graceful graph satisfied Radio analytic mean condition as follows

ie) d(u,v) + $\left\lceil \frac{|\theta(u)^2 - \theta(v)^2|}{2} \right\rceil \ge$ 1+ diam (G) ………………(6)

f(u)= 4n+1 , f(u_i')= n+3+i , $0 \le i \le n-2$, f(u_i') =4n , i=6 , f(u_i') = f(u_i')-1 , i=6 , f(w) =0 , f(w_i')= n-4+i , $0 \le i \le n-1$.

Case (i) consider the distance between any two vertices if d(u,v) = k if k=1

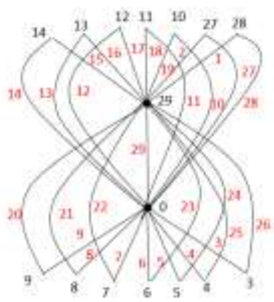
1) View the pair (u,w) , d(u,w) + $\left\lceil \frac{|(4n+1)^2 - (0)^2|}{2} \right\rceil \ge 3$
2) View the pair (u , u_i') , d(u , u_i')+ $\left\lceil \frac{|(4n+1)^2 - (n+3+i)^2|}{2} \right\rceil \ge 3$
3) View the pair (w, w_i') , d(w, w_i')+ $\left\lceil \frac{|(0)^2 - (n-4+i)^2|}{2} \right\rceil \ge 3$
4) View the pair (w, u_i') , d(w, u_i') + $\left\lceil \frac{|(0)^2 - (n+3+i)^2|}{2} \right\rceil \ge 3$
5) View the pair (u, w_i') , d(u, w_i') + $\left\lceil \frac{|(4n+1)^2 - (n-4+i)^2|}{2} \right\rceil \ge 3$

Case(ii) consider the distance between any two vertices if d(u,v) = k if k= 2

1) View the pair (u_i', u_j') , d (u_i', u_j') + $\left\lceil \frac{|(n+3+i)^2 - (n+3+j)^2|}{2} \right\rceil \ge 3$
2) View the pair (w_i', w_j') , d(w_i', w_j') + $\left\lceil \frac{|(n-4+i)^2 - (n-4+j)^2|}{2} \right\rceil \ge 3$

Vertex labels	Edge labels
f (w, u_i')	E(G) =$\sum_{i=1}^{n-2} n + 2 + i$
f (w, u_i')	E(G) =$\sum_{i=0}^{1} 4n - i$
f(u , u_i')	E(G) =$\sum_{i=1}^{n-2} 2n + i$
f(u , u_i')	E(G) =$\sum_{i=0}^{1} 1 + i$
f(w , w_i')	E(G) =$\sum_{i=0}^{n-1} n + 2 - i$
f(u , w_i')	E(G) =$\sum_{i=1}^{n-1} 2n + 6 + i$
f(w, w_i')	E(G) =$\sum_{i=1}^{n-5} n - 5 + i$

Illustration 10:



ie) Ramgn is B^2(7,7)= 29

Theorem 3.7 : The Radio Analytic mean graceful number of D_2 ($B_{n,n}$) = 8n +n –1, n ≥ 5

Proof :

Let u,v,w,t are the centre of vertices in D_2 ($B_{n,n}$) .Let u_i' ,v_i' , w_i', t_i' are the pendent vertices of u,v,w,t. we define the vertex labelling v(G)= $\{0,1,2,3 \ldots\ldots p\}$ and edge labelling E(G) = $\{1,2,3 \ldots\ldots q\}$ are distinct. Therefore D2 ($B_{n,n}$) is graceful graph. In this graceful graph satisfied the Radio analytic mean condition

ie) d(u,v) + $\left\lceil\frac{|\theta(u)^2-\theta(v)^2|}{2}\right\rceil$ ≥ 1+ diam (G) ………………(7)

The D_2 (Bn,n) having order of the vertices V(G) = 4n+4 and order of the edges E(G) = 8n +4 . its diameter of 3.Clearly f is bijective function .

we define the vertex labelling f: V(B_2 (n,n)) $\rightarrow z^+$

f(u)= 0 ,

f(u_i') = 8n +n –i , 1≤ $i \leq n$,

f(v)=n , f(v_i') = 6n +i , 0≤ $i \leq n-1$,

f(w) =5n-1 ,

f(w_i') = n +(3+j) , 0≤ $j \leq 1$, i= 3,4 ,

f(w_i') = 1 , i= 1 , f(w_i') =n- 1 , i=2 ,

f(t_i') = 2n + 3 , 1≤ i

f(t_i') = 3n + j , 1≤ $j \leq 2$

f(t)= 4n+2

f(w_i') = 2n+2 , i= 5

f(t_j') = 4n+i , 1≤ $i \leq 2$, 4≤ $j \leq 5$

we are indicate to distinct edge labels listed below :

f(u,v)	Distinct edge labels
f(u, v_i')	E(G) = $\sum_{i=0}^{n-1} 6n + i$
f(u, u_i')	E(G) = $\sum_{i=1}^{n} 9n - i$
f(v, u_i')	E(G) = $\sum_{i=1}^{n} 8n - i$
f(v, v_i')	E(G) = $\sum_{i=1}^{n} 6n + i - i$
f(w, w_i')	E(G) = $\sum_{i=1}^{n-3} 2n + 4 + i$
f(w, w_i'),i=3	E(G) = 4n,
f(w, w_i') , i=1	E(G) = 4n+3
f(w, t_i'), 1≤ $i \leq n$	E(G) = 2n+1 ,n+3,n+2,n-1,n-2
f(t, t_i'), 1≤ $i \leq n$	E(G) = n+4 ,n+1,n,n-3, n-4
f(t, w_i'), 1≤ $i \leq n$	E(G)= 2n , 2n+3 ,2n+4 ,3n+3, 4n+1
$f(t,v), f(t,v), f(v,w), f(u,w)$	E(G)= 4n+2, 3n+2, 4n-1,5n-1

Case(i) : consider the double copies of D_2 ($B_{n,n}$) graceful graph Distance between any two vertices is 1. ie) d (u,v)= k ,k =1. We shows that each pair of nodes fulfil the Radio analytic mean condition one by one.

(i) View the pair d(u ,u_i') + $\left\lceil\frac{|(0)^2-(8n+n-i)^2|}{2}\right\rceil$ ≥4

(ii) View the pair d(u ,v_i') + $\left\lceil\frac{|(0)^2-(6n+i)^2|}{2}\right\rceil$ ≥4

(iii) View the pair d(v,v_i') + $\left\lceil\frac{|(n)^2-(6n+i)^2|}{2}\right\rceil$ ≥4

(iv) View the pair d (v,w) + $\left\lceil\frac{|(n)^2-(5n-1)^2|}{2}\right\rceil$ ≥4

(v) View the pair d (u,t) + $\left\lceil\frac{|(0)^2-(4n+2)^2|}{2}\right\rceil$ ≥4

(vi) View the pair d (v,t) + $\left\lceil\frac{|(n)^2-(4n+2)^2|}{2}\right\rceil$ ≥4

(vii) View the pair d (t,t_i') + $\left\lceil\frac{|(4n+2)^2-(2n+3)^2|}{2}\right\rceil$ ≥4

(viii) View the pair d (w,w_i') + $\left\lceil\frac{|(5n-1)^2-(n+(3+j))^2|}{2}\right\rceil$ ≥4

Case (ii) : consider the double copies of D_2 ($B_{n,n}$) in graceful graph Distance between any two vertices is 2. ie) d (u,v)= k ,k =2. We shows that each pair of nodes fulfil the Radio analytic mean condition one by one.

(i) View the pair $d(v_i', v_j') + \left\lceil \frac{|(6n+i)^2-(6n+j)^2|}{2} \right\rceil \geq 4$

(ii) View the pair $d(u_i', u_j') + \left\lceil \frac{|(8n+n-i)^2-(8n+n-j)^2|}{2} \right\rceil \geq 4$

(iii) View the pair $d(t_i', t_j') + \left\lceil \frac{|(3n+j)^2-(4n+i)^2|}{2} \right\rceil \geq 4$

Case(iii) : consider the distance between any pair two vertices is 3 ie) d(u,v) =k if k= 3. We verified that all the cases in graceful graph satisfied the Ramc (f).

(i) View the pair $d(v_i', t_i') + \left\lceil \frac{|(6n+i)^2-(3n+j)^2|}{2} \right\rceil \geq 4$, where $1 \leq i \leq n$

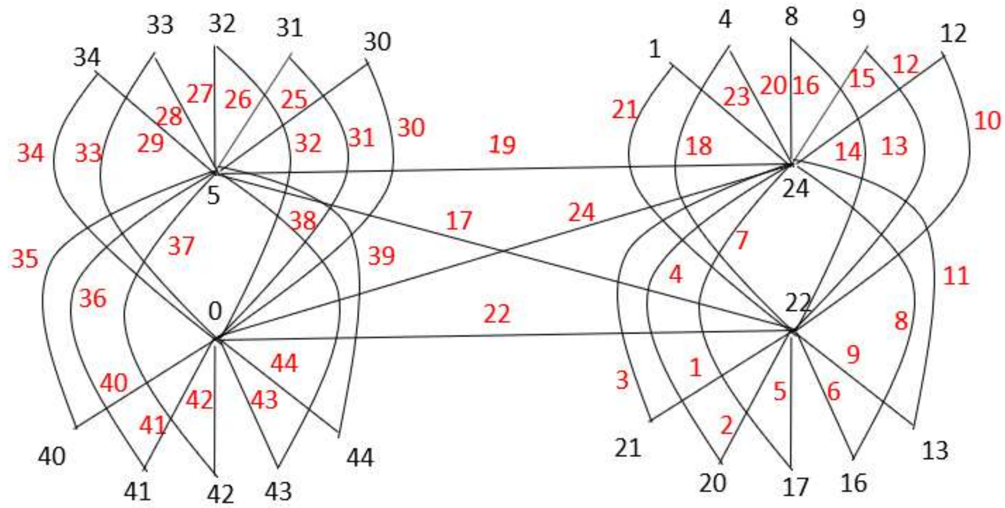
(ii) View the pair $d(u_i', w_i') + \left\lceil \frac{|(8n+n-i)^2-(n+(3+j))^2|}{2} \right\rceil \geq 4$, where $1 \leq i \leq n$

(iii) View the pair $d(v_i', w_i') + \left\lceil \frac{|(6n+i)^2-(n+(3+j))^2|}{2} \right\rceil \geq 4$, where $1 \leq i \leq n$

(iv) View the pair $d(u_i', t_i') + \left\lceil \frac{|(8n+n-i)^2-(3n+j)^2|}{2} \right\rceil \geq 4$, where $1 \leq i \leq n$

Here all the cases of graceful graph satisfied the inequalities conditions by (1) considering in the above cases.

Illustration 11:



ie) D_2 ($B_{5,5}$) =44

Theorem 3.8 : The Radio analytic Mean graceful number of friendship graph $Fl_n = 3n$, $n \geq 5$

Proof:

Let w be the centre vertex of graceful friendship graph. All the pendent vertices are connected to triangle shape. All the triangle shape are mapping to w .The order of the vertex set V(G) = 2n+1 and order of the edge set E(G) = 3n . we define the edge label E = $\{ww_i', 0 \leq i \leq n\}$. Its diameter of the friendship's graph is 2. We verified that Fl_n satisfied the radio analytic mean condition.

ie) $d(u,v) + \left\lceil \frac{|\theta(u)^2-\theta(v)^2|}{2} \right\rceil \geq 1+$ diam (G) ………………(8)

we define the bijective map f: V(Fl_n) → z^+ .

f(w) = 3n , $f(w_i')$= i , $0 \leq i \leq n+1$,$f(w_{n+2}')$= n+4 , $5 \leq n \leq 6$, $f(w_{n+5}')$= 3n-1 ,if n=5

(i)Verify the pair (w , w_i') , $0 \leq i \leq n$

$d(w, w_i') + \left\lceil \frac{|(3n)^2-(i)^2|}{2} \right\rceil \geq 3 \Rightarrow 1 + \left\lceil \frac{|(15)^2-(2)^2|}{2} \right\rceil \geq 3$

(ii) Verify the pair (w , w_i') , 0≤ i ≤ n

d(w , w_i') + $\left\lceil \frac{|(3n)^2-(i)^2|}{2} \right\rceil \geq 3 \Rightarrow 1 + \left\lceil \frac{|(15)^2-(10)^2|}{2} \right\rceil \geq 3$

(iii) Verify the pair (w, w_{n+2}') , 5≤ n ≤ 6

d(w , w_{n+2}') + $\left\lceil \frac{|(3n)^2-(n+4)^2|}{2} \right\rceil \geq 3 \Rightarrow 1 + \left\lceil \frac{|(15)^2-(9)^2|}{2} \right\rceil \geq 3$

(iv) Verify the pair (w, w_{n+5}') , 5≤ n ≤ 6

d(w , w_{n+5}') + $\left\lceil \frac{|(3n)^2-(3n-1)^2|}{2} \right\rceil \geq 3 \Rightarrow 1 + \left\lceil \frac{|(15)^2-(14)^2|}{2} \right\rceil \geq 3$

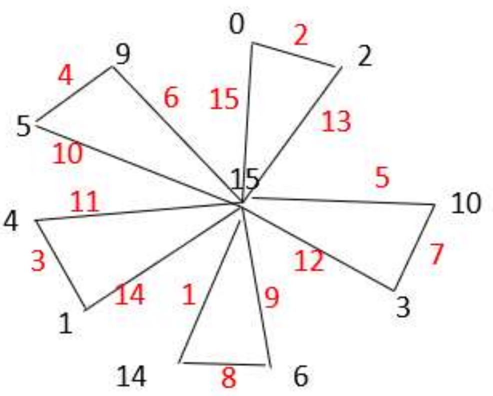
(v) Verify the pair (w_i' , w_{n+5}') , 0≤ i ≤ n+1

d(w_i' , w_{n+5}') + $\left\lceil \frac{|(i)^2-(3n-1)^2|}{2} \right\rceil \geq 3 \Rightarrow 2 + \left\lceil \frac{|(3)^2-(14)^2|}{2} \right\rceil \geq 3$

(vi) Verify the pair (w_{n+2}', w_{n+5}') , 5≤ n ≤ 6

d(w_{n+2}', w_{n+5}')+ $\left\lceil \frac{|(n+4)^2-(3n-1)^2|}{2} \right\rceil \geq 3 \Rightarrow 2 + \left\lceil \frac{|(9)^2-(14)^2|}{2} \right\rceil \geq 3$

Illustration 12 :



ie) ramgn is (Fl$_5$) = 15

Conclusion: In this paper we investigate radio analytic mean graceful number of star graph, fan graph, friendship graph and Bistar related graphs. Further we have to extend in Radio analytic edge odd graceful path & cycle related graphs and network related graphs.

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