A MATHEMATICAL APPROACH TO MANAGING WAIT TIMES IN HEALTHCARE FACILITIES USING QUEUING THEORY

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ABSTRACT
Efficient management of patient wait times in healthcare facilities is critical for ensuring patient satisfaction, reducing operational costs, and improving overall healthcare delivery. Queuing theory, a mathematical framework for analyzing waiting lines, provides a robust method to model, analyze, and optimize healthcare service systems. This paper explores the application of queuing theory in healthcare, focusing on strategies to reduce wait times, allocate resources effectively, and predict patient flow. By examining various queuing models, including single-server, multi-server, and network queuing systems, this research highlights their practical applications in different healthcare scenarios such as emergency departments, outpatient clinics, and diagnostic centers. The findings underscore the potential of mathematical modeling in enhancing decision-making, improving resource utilization, and delivering timely care in the healthcare sector.

INTRODUCTION

In hospital systems, achieving the goals of increasing resource utilization to reduce costs, minimizing patient wait times for timely care, and enhancing patient satisfaction often presents conflicting challenges. Queuing models have become a popular tool among researchers and system designers as they offer relatively accurate assessments of system performance, are analytical in nature, and provide quick solutions for "what-if" analyses.

A significant body of research explores the use of queuing models in analyzing and designing hospital environments. This paper reviews and classifies the existing literature to encourage further research into the application of queuing models in healthcare. Effective planning is critical when establishing a new hospital. Proper planning ensures the smooth operation of systems, whereas inadequate planning can lead to failure. The planning process begins with defining the hospital's goals, as clear objectives provide direction and focus. This is followed by assessing the organization's external environment and the internal and external resources available to achieve these goals. This comprehensive approach helps identify cost-effective strategies for meeting objectives.

Queuing models are widely applied across various healthcare systems, including hospitals (Cochran and Roche, 2009), the pharmaceutical industry (Viswanadham and Narahari, 2001), and organ transplantation (Zenios, 1999). Prater (2001) compiled a bibliography of medical queuing applications, which remains a valuable resource. This paper expands on prior work by reviewing and classifying queuing model applications in hospital management, with a particular focus on the optimal allocation of beds in different hospital wards.

The process begins with estimating the required capacity of the system, which involves separating demand into two components: controllable and uncontrollable. Controllable demand includes patients requiring scheduled appointments, while uncontrollable demand pertains to emergency cases.

The history of queuing theory spans nearly a century, with foundational contributions such as the Chapman-Kolmogorov equation describing statistical equilibrium. Significant developments in the field include "Rational Determination of the Number of Circuits" by Brookmeyer et al. (1960), which addressed optimization in queuing systems, and early economic models by Hiller (1963) and Heymans (1968). These studies laid the groundwork for incorporating optimization techniques into queuing theory, including the M/M/1 and M/G/1 systems.

Research on queuing systems with impatient customers has also gained attention. For example, Udagawa and Nakamura (1957) developed models examining resistance and abandonment behaviors in M/M/1 systems. Ancker and Gafarian (1962, 1963) extended this work to consider blocking and reneging behaviors in both M/M/1 and M/M/R systems.

Since then, operational researchers have advanced the field significantly, applying mathematical optimization techniques to model more complex queuing scenarios. These developments highlight the potential for queuing models to enhance the design and operation of healthcare systems.

In this system, incoming customers are blocked with probability n/N and therefore have a chance of joining the system.

$E_n = 1 - n/N \quad (n=0, 1, 2, 3 \ldots N)$

Where n is the number in your system and N is the maximum number allowed in your system. After entering the queue, each customer waits a certain amount of time for service. This is the density function random variable.

$d(t) = \alpha e^{-\alpha t}$

If the service is not started by then, it will leave the system and be lost. Ancker and Gafarian (1963) **[9]** considered another model. The model makes the same assumptions about abandonment behaviors, with the difference being that abandonment behaviors are more likely to involve inbound customers in the system.

$E_n = \{\beta/n\}$ if $(0 \leq \beta \leq 1, n = 1, 2, 3 \ldots)$

If $e_n = 1$ $(n = 0)$

Where 'n' is the number of systems and β is a measure of customer willingness to queue. This issue removed the limit on the number of customers in the system. The above work on this topic is limited to special cases where blocking and denial depend on the number of customers in the system. Adam et al. (2001) **[10]** deal extensively with multiqueue queues. We have thus outlined the growth of queuing theory and identified important developments and directions. Seguin (2020) conducted research at his Adekunle Ajasin University in Akungba-Akoko, Nigeria, on performance modeling of health service delivery using queuing theory. The purpose of this study was to model an appropriate queuing system by determining patient waiting, arrival, and service times in an AUAA health environment and validating the model using simulation techniques bottom. The study was conducted at his AUAA Health Center Akungba Akko. A suitable model was developed using analytical and simulation methods. A stopwatch was used to calculate the number of minutes each patient spent arriving and picking up a ticket or registering from the reception area to the final area (examination room area). Data on each patient's arrival time, waiting time, and service time were collected on weekdays.

(Monday to Friday) for her 3 weeks. Data were calculated and analyzed using Microsoft Excel. Based on the analyzed data, a queuing system for real patient situations was modeled and simulated with PYTHON software. The results obtained from the simulation model showed that the average patient arrival rate on Friday of week 1 was lower than the average patient service rate (that is, 5.33 > 5.625 ($\lambda > \mu$)). In other words, there is a queue that goes on forever. Service facilities are manned at all times. After analyzing the AAUA health center system as a whole, we found that when the system was very busy, queue lengths increased. The study recommends that the AUAA health center should improve the quality of service to patients who visit the health center.

OBJECTIVE OF RESEARCH WORK

This research has two primary objectives:

1. To examine patient wait times at hospitals and identify operational challenges that may contribute to delays. The waiting experience significantly influences patients' perception of service quality.
2. To focus on patients requiring emergency services, such as those needing intensive care, maternity, or treatment for accidents.

QUEUING THEORY CONCEPT

Queues represent a significant challenge for healthcare systems worldwide. In today's healthcare environment, a great deal of research is being conducted to optimize queuing systems, particularly within hospital settings, in an effort to improve the efficiency and quality of patient care. These studies explore various techniques to streamline patient flow, reduce wait times, and enhance overall operational efficiency. However, despite the advancements in research and implementation of queuing strategies in developed countries, such progress remains largely absent in many developing nations.

The purpose of this paper is to explore the queuing conditions within public hospitals in developing countries and offer insights into potential strategies for improving decision-making processes within these healthcare institutions. By examining the current state of queuing systems and their impact on hospital operations, this paper aims to provide practical recommendations that could help enhance the efficiency and effectiveness of healthcare delivery.

One of the most powerful tools available for analyzing queuing systems is queuing theory. This mathematical approach is commonly used to model and evaluate the performance of queuing systems, providing valuable insights into key performance parameters such as wait times, service times, and the number of patients being served within a given timeframe. Queuing theory has proven to be particularly beneficial in healthcare settings, where long wait times and inefficiencies in patient flow can have significant consequences for both patient outcomes and hospital operations. By applying queuing theory to hospital environments, healthcare managers can make data-driven decisions that improve resource allocation, minimize delays, and optimize patient care.

In developed countries, queuing theory has become an essential tool for healthcare management, particularly in the context of hospital operations. With the help of advanced mathematical models, hospitals in these regions can better understand their queuing dynamics, identify bottlenecks in patient flow, and implement targeted strategies to alleviate delays. For example, queuing theory can be used to determine the optimal number of staff required at various times of day, the most efficient way to allocate resources across departments, and the best ways to manage patient appointments and waiting lists. These insights allow hospitals to make more informed decisions and improve the overall patient experience.

Unfortunately, many developing countries lack the resources, infrastructure, and expertise necessary to apply queuing theory and other advanced management tools in their healthcare systems. As a result, public hospitals in these regions often struggle with overcrowding, long wait times, and inefficient use of resources. In these contexts, patients may experience longer delays in receiving care, which can have serious consequences for their health and well-being. Moreover, hospital administrators may face difficulties in making informed decisions regarding resource allocation and operational improvements, further exacerbating the challenges faced by these institutions.

This paper seeks to address this gap by analyzing the queuing conditions in public hospitals in developing countries and offering practical recommendations for improving decision-making processes. By leveraging the principles of queuing theory, hospitals in these regions can better understand their operational inefficiencies, identify areas for improvement, and implement targeted interventions that improve patient care and hospital performance. This approach holds the potential to create meaningful improvements in healthcare delivery, even in resource-limited settings, ultimately contributing to the development of more effective healthcare systems in the global context.

5. Classification of queuing models

Using Kendal & Lee notations Generally, any queuing models may be completely specified in the following symbolic form:

A / B/ C/ D / E
A → Type of distribution of inter – arrival time
B → Type of distribution of inter – service time
C → Number of servers
D → Capacity of the system
E → Queue discipline
M→Arrival time follows Poisson distribution and service time follows an exponential distribution.

Model I: M / M / 1: ∞ / FCFS
Where M -Arrival time follows a Poisson distribution
M →Service time follows a exponential distribution
1 → Single service model
∞→ Capacity of the system is infinite
FCFS → Queue discipline is first come first served
Model II: M / M / 1: N / FCFS
Where N →Capacity of the system is finite
Model III: M / M / 1: ∞ / SIRO
Where SIRO → Service in random order
Model IV: M / O / 1: ∞ / FCFS
Where D → Service time follows a constant distribution or is deterministic
Model V: M / G / 1: ∞ / FCFS
Where G → Service time follows a general distribution or arbitrary distribution
Model VI: M / Ek / 1: ∞ / FCFS
Where Ek → Service time follows Erlangen distribution with K phases.
Model VII: M / M / K: ∞ / FCFS
Where K → Multiple Server model
Model VIII: M / M / K: N / FCFS
Model I: M / M /1: ∞ / FCFS

6. Formulas:

1. Utilization factor traffic intensity / Utilization parameter /Busy period
$$\rho = \lambda / \mu$$
Where λ = Mean arrival rate, μ= mean service rate
Note: $\mu > \lambda$ in single server model only
2. Probability that exactly zero units are in the system
$$Po = 1 - \lambda / \mu$$
3. Probability that exactly 'n' units in the system
$$Pn = p_0 (\lambda / \mu)n$$
4. Probability that n or more units in the system
$$Pn \text{ or more} = (\lambda / \mu)n$$
More than 'n' means n should be n+1
5. Expected number of units in the queue / queue length
$$L q = \lambda 2 / \mu (\mu - \lambda)$$
6. Expected waiting time in the queue

$$W_q = L_q / \lambda$$

7. Expected number of units in the system

$$L = L_q + \lambda / \mu$$

8. Expected waiting time in the system

$$W = W_q + 1/\mu$$

9. Expected number of units in queue that from time to time – (OR) non - empty queue size

$$D = \mu / \mu - \lambda$$

10. Probability that an arrival will have to wait in the queue for service

$$\text{Probability} = 1 - P_o$$

11. Probability that an arrival will have to wait in the queue more than w (where w > o), the waiting time in the queue

$$\text{Probability} = (\lambda/\mu)\, e\, (\lambda - \mu)\, w$$

12. Probability that an arrival will have to wait more than v (v > o) waiting time in the system is

$$= e\, (\lambda - \mu)\, v$$

13. Probability that an arrival will not have to wait in the queue for service = Po

Model 1 - Problems

Example: In a municipality hospital patient's arrival are considered to be Poisson with an arrival interval time of 10mins. The doctors (examination and dispensing) time many be assumed to be ED with an average of 6mins find:

a) What is the chance that a new patient directly sees the doctor?
b) For what proportion of the time the doctor is busy?
c) What is the average number of patients in the system?
d) What is the average waiting time of the system?
e) Suppose the municipality wants to recruit another doctor, when an average waiting time of an arrival is 30mins in the queue. Find out hose large should be to justify a 2nd doctor?

Solution:

$$\lambda = 1/10 \times 60 = 6/hr$$
$$\mu = 1/6 \times 60 = 10/hr$$

Probability that a new patient straight away sees the doctor:

$$P_0 = 1 - \lambda/\mu = 1 - 6/10 = 0.4$$

a) Proportion of time the doctor is busy:

$$\rho = \lambda/\mu = 6/10 = 0.6hr$$

b) Average number of patients in the system:

$$L = L_q + \lambda/\mu = \lambda^2/\mu(\mu-\lambda) + \lambda/\mu = 6^2/10(10-6) + 6/10 = 36/40 + 6/10$$
$$L = 1.5$$

(c) Average waiting in the system:

$$W = W_q + 1/\mu = L_q/\lambda + 1/\mu = \lambda^2/\mu(\mu-\lambda)\lambda + 1/\mu$$
$$W = 6/10(10-6) + 1/10 = 6/40 + 1/10 = 0.25$$
$$Wq = 30/60 = 0.5 \text{ hr}$$
$$Wq = L_q/\lambda = \lambda^2/\mu(\mu-\lambda)\lambda = \lambda/\mu(\mu-\lambda) = 6/10(10-6) = 0.5$$

(d) $\lambda/10(10-\lambda) = 0.5$

$\lambda/100-10\lambda = 0.5$

$\lambda = 100-10\lambda \times 0.5$

$\lambda = 50-5\lambda$

$\lambda = 50/6 = 8.33/ \text{ hr}.$

λ The value of has to be increased from 6 to 8.33 justify a second doctor.

CONCLUSION

In our study, the system not only serves people who request services in hospitals, but also uses their time for other activities. The benefits of using this open source can extend to society as a whole. He is not just one hospital that can benefit from the current system design and setup, he can also serve multiple hospitals at the same time. A single hospital can track its own queues with specific power users as queue managers. This design allows cost-sharing agreements between hospitals without spending budget on additional development. Multiserver queues can be used to estimate average wait times, queue lengths, number of servers, and service rates**.**

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