An Estimate Of The Finite Population Mean Using An Exponential Type

R. Mithili [1,2] and Bharti V. Nathwani [2,*]

[1] *Department of Humanities and Applied Sciences, Dilkap Research Institute of Engineering and Management Studies, Neral, Maharashtra, India.*

[2] *Amity School of Applied Sciences, Amity University Maharashtra, Mumbai, Panvel - 410206, Maharashtra, India*

**Corresponding Author*

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ABSTRACT:

In this article, the work estimates the population mean of the SRSWOR for the studied variable using an improved family of exponential estimators employing some well-known population parameters for the auxiliary variable. First-order approximations were used to derive the formula for the bias and mean square error (MSE) of the proposed family of estimators. With the exponential family of estimating, a comparison has been done. Through the empirical analysis, an enhancement to the family of estimators has been demonstrated.

1. Introduction

In all areas of life, numerous applications given the importance of sampling theory in statistics. The use of sampling theory extends beyond sample selection to include data about the population where its parameters are found. Sampling theory's primary goal is to estimate the population parameter. Accuracy of the estimator increases with the use of an auxiliary variable. In survey sampling, lowering the variance of the estimators of the population parameters increases the precision to an estimators of the mean of the study variable with the auxiliary variables. Since the best estimator of any population parameter is its corresponding statistic, the sample mean is the most appropriate estimator for the estimating a population mean[8]. Many authors have used auxiliary variables and confirmed with calculation the improvement in efficiency of estimators.Use of auxiliary information at the estimation stage was established from the work done by Watson (1937), Cochran (1940), Robson (1957) and Murthy (1964). The improved proposed estimators using auxiliary information for ratio and product type were studied by [2],[3], [11],[4], [6], [9], [10], [13], [14].

Notations

Let $\bar{Y}$ and $\bar{X}$ be the population mean of the study and auxiliary variable respectively. Let $\bar{y}$ and $\bar{x}$ be the sample means of the study and auxiliary variable respectively. To obtain the bias and the mean square error (MSE) of the estimators, let $\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$
such that

$$E(e_i) = 0, i = 0,1$$
$$E(e_0^2) = \theta C_y^2$$
$$E(e_1^2) = \theta C_x^2$$
$$E(e_0 e_1) = \theta C_{yx} = \theta\rho C_y C_x$$

where $C_y^2 = \frac{S_y^2}{Y^2}, C_x^2 = \frac{S_x^2}{X^2}, \theta = \frac{N-n}{Nn}$

Estimators in literature

The existing estimators in the sampling literature are covered in this section. As Nursel Koyuncu and Cem Kadilar [5], estimator of the population mean $\bar{Y}$ of the variate of the study, the use of information about the population proportion have certain attribute.

$$\bar{t} = \bar{y}\left(\frac{(a\bar{X} + b)}{\alpha(a\bar{x} + b) + (1 - \alpha)(a\bar{X} + b)}\right)^g \quad (1)$$

where a$(\neq 0)$ and b can be either functions or real integers that represent the known parameters of the auxiliary variable x , such as the population's correlation coefficient (ρ), skewness $(\beta_1(x))$, kurtosis$(\beta_2(x))$, standard deviation (S_x), and coefficient of variation (C_x). Here, g and α are appropriately designed scalars such that the t mean square error is minimised as possible.

The expressions of MSE of the estimators is

$$MSE(\bar{t}) = \theta\bar{Y}^2\left[C_y^2 + \alpha^2 v^2 g^2 C_x^2 - 2\alpha v g C_{yx}\right] \quad (2)$$

A family of estimators based on a known parameter connected to auxiliary data was proposed by Koyuncu and Kadilar([4])

$$\bar{Y}_k = \left[w_1\bar{y} + w_2\left(\frac{\bar{x}}{\bar{X}}\right)\right]^g \left[\exp\left(\frac{\eta(\bar{X} - \bar{x})}{\eta(\bar{X} + \bar{x}) + 2\lambda)}\right]\right] \quad (3)$$

The MSE of these estimators are

$$\text{MSE}(\bar{Y}_k) = \bar{Y}^2 A w_1^2 + B w_2^2 + \bar{Y}^2 D w_1 + \bar{Y} G w_2 + \bar{Y}^2 + \bar{Y} w_1 w_2 F \quad (4)$$

where,
$\text{A} = 1 + \theta\left[C_y^2 - 2\eta C_{yx} + \eta^2 C_x^2\right], \text{B} = 1 + (g^2 + \eta^2 + g(g-1) - 2g\eta)\theta C_x^2,$
$\text{D} = \eta\theta C_{yx} - 2 - \frac{3}{4}\eta^2\theta C_x^2, \text{G} = \eta g - \frac{3)}{4}\eta^2 - \eta(\eta - 1)\theta C_x^2 - 2,$

$\mathrm{F} = 2 + 2(g-\eta)\theta C_{yx} + (2\eta^2 + g(g-1) - 2g\eta)\theta C_x^2$
The expression of minimum mean square error of the estimator is

$$MSE_{\min}(\bar{Y}_k) = \bar{Y}^2\left[1 - \frac{BD^2 - DFGAG^2}{4BA - F^2}\right] \quad (5)$$

The proposed estimator of Rajesh Singh et al.,2020([12]) the modified exponential estimator as

$$\bar{\theta}_{PExpi} = \bar{y}\left(w_1 + w_2\frac{\bar{X}}{\bar{x}}\right)\exp\frac{\alpha(\bar{X}-\bar{x})}{\alpha(\bar{X}+\bar{x})+2\beta}, (i = 1,2,3,4) \quad (6)$$

For $[i = 1, \alpha = 1, \beta = -1], [i = 2, \alpha = 1, \beta = 0], [i = 3, \alpha = 1, \beta = 1], [i = 4, \alpha = 0, \beta = 1]$
The mean square error of the estimator is

$$MSE\left(\bar{\theta}_{PExpi}\right) = \bar{Y}^2[1 + Aw_1^2 + Bw_2^2 - 2Cw_1 - 2Dw_2 + 2Ew_1w_2] \quad (8)$$

where,
$A = 1 + \theta(C_Y^2 + 4\lambda^2 C_X^2 - 4\lambda C_{YX}), B = 1 + \theta(C_Y^2 + (4\lambda^2 + 4\lambda + 3)C_X^2 - 4(\lambda+1)C_{YX}),$
$C = 1 + \theta\left(\frac{3}{2}\lambda^2 C_X^2 - \lambda C_{YX}\right), D = 1 + \theta\left(\left(\frac{3}{2}\lambda^2 + \lambda + 1\right)C_X^2 - (\lambda+1)C_{YX}\right)$
$E = 1 + \theta(C_Y^2 + (4\lambda^2 + 2\lambda + 1)C_X^2 - 2(2\lambda+1)C_{YX})$
The minimum mean square error of the estimator is

$$\mathrm{MSE}\left(\bar{\theta}_{PExpi}\right)_{\min} = \bar{y}^2\left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \quad (9)$$

Table-1

(Existing estimators of literature[1])

s.no.	Estimators	min. MSE
1	$T_u = \bar{y}$	$V(\bar{y}) = \theta\bar{Y}^2 C_Y^2$
2	$T_r = \bar{y}\bar{X}$, Cochran	$MSE(T_r) = \theta\bar{Y}^2(C_Y^2 + C_X^2 - 2\rho C_Y C_X)$
3	$T_P = \bar{y}\bar{\bar{x}}/\bar{X}$, Murthy	$MSE(T_P) = \theta\bar{Y}^2\left(C_y^2 + C_x + 2\rho C_y C_x\right)$
4	$T_{1-\text{rat}} = \bar{y}\exp\ \overline{p\bar{X} - \bar{x}\bar{\bar{x}}}$, Bahl and Tujeta	$MSE(T_{1-\text{rat}}) = \theta\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x\right)$
5	$T_{2-\text{pro}} = \bar{y}\exp\frac{\bar{x}-\bar{x}}{\bar{X}+\bar{x}}$, Bahl and Tujeta	$MSE\left(T_{1-\text{pro}}\right) = \theta\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4} + \rho C_y C_x\right)$

6	$T_M = 2^{-1}\bar{y}\left[\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} + \exp\left(\frac{X-\bar{x}}{\bar{X}+\bar{x}}\right)\right]$, Yunusa et al.,	$MSE(T_M) = \theta\bar{Y}^2 C_y^2(1-\rho^2)$

Proposed Estimators

By the motivation of Rajesh Singh (2020) and Koyuncu and Kadilar (2009), the proposed estimator of exponential ratio type estimators to estimate the parameter of the population is

$$t_{mi} = \bar{y}\left(w_1 + w_2\frac{\bar{X}}{\bar{x}}\right)\left(\frac{(a\bar{X}+b)}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}\right)^{g} \tag{10}$$

where w_1 and w_2 are scalars with $w_1 + w_2 = 1$ and a$(\neq 0)$, b are either real numbers or functions of the known parameters of the auxiliary variable x, such as standard deviation (S_x), coefficient of variation (C_x), correlation coefficient (ρ) of the population. Also, g , α are suitably chosen scalars such that the mean square error of t_{mi} is minimum. we express equation (1.9) in terms of e 's to obtain,

$$t_{mi} = \bar{Y}(1+e_0)(w_1 + w_2 - w_2 e_1 + w_2 e_1^2)(1+\alpha e_1 v)^{-g} \tag{11}$$

Multiplying and neglecting the higher-order terms of e 's which are greater than two powers in equation (1.10), we have

$$t_{mi} = \bar{Y}\left[(w_1 + w_2) + (w_1 + w_2)\left(-\alpha e_1 v g + e_0 - \alpha e_0 e_1 v g + \frac{g(g+1)}{2}\alpha^2 e_1^2 v^2\right)\right]$$

Subtracting $\bar{Y}$ both side,

$$t_{mi} - \bar{Y} = \bar{Y}[(w_1 + w_2 - 1) + (w_1 + w_2)(-\alpha e_1 v g + e_0 - \alpha e_0 e_1 v g + \frac{g(g+1)}{2}\alpha^2 e_1^2 v^2\Big) + w_2(-e_1 + e_1^2)(1 - \alpha e_1 v g$$

Taking expectation in equation (1.12) gives the
BIAS $(t_{mi}) =$

$$E(t_{mi} - \bar{Y}) = \bar{Y}\left[(w_1 + w_2 - 1) + (w_1 + w_2)\left(-\alpha v g \rho\theta C_y C_x + \frac{g(g+1)}{2}\alpha^2 v^2 \theta C_x^2\right)\right]$$

To obtain the mean square error (MSE) of the estimators under SRSWOR, squaring equation (1.12) both side and neglecting higher powers of e 's, we get

$$(t_{mi}-\bar{Y})^2 = \Big(\bar{Y}[(w_1+w_2-1)+(w_1+w_2)(-\alpha e_1 vg + e_0 - \alpha e_0 e_1 vg + \frac{g(g+1)}{2}\alpha^2 e_1^2 v^2\Big) + w_2(-e_1+e_1^2)(1-\alpha e_1 vg$$

$$(t_{mi}-\bar{Y})^2 = \bar{Y}^2[(w_1^2+w_2^2+1-2w_1-2w_2w_1w_2)+(w_1^2+w_2^2+2w_1^2)\times(e_0^2+\alpha^2v^2e_1^2g^2-2\alpha vge_0e_1) + w_2^2e_1^2 + 2(w_1^2+2w_1w_2+w_2^2-w_1-w_2)(e_0-\alpha vge_1-\alpha vge_0e_1 + \frac{g(g+1)}{2}\alpha^2v^2e_1^2\Big) + 2(w_2^2+w_1w_2-w_2)(-e_1+e_1^2+\alpha e_1^2 vg - e_0e_1)$$

Taking Expectation both side,

$$MSE(t_{mi}) = \bar{Y}^2[1 + Aw_1^2 + Bw_2^2 - 2Cw_1 - 2Dw_2 + 2Ew_1w_2] \tag{17}$$

where,

$A = 1 + \theta C_Y^2 - 4\alpha vg\theta\rho C_Y C_X + \alpha^2 v^2 \theta C_X^2 (2g^2 + g)$

$B = 1 + \theta C_Y^2 - 4(\alpha vg + 1)\theta\rho C_Y C_X + \theta C_X^2(\alpha^2v^2g^2 + 3 + g(g+1)\alpha^2v^2 + 4\alpha vg)$

$C = 1 - \alpha vg\rho\theta C_Y C_X + \frac{g(g+1)}{2}\alpha^2v^2\theta C_X^2$

$D = 1 - (1+\alpha vg)\theta\rho C_Y C_X + \alpha vg\theta C_X^2 + \frac{g(g+1)}{2}\alpha^2v^2\theta C_X^2 + \theta C_X^2$

$E = 1 + \theta C_Y^2 - 4\alpha vg\theta\rho C_Y C_X - 2\theta\rho C_Y C_X + \theta C_X^2(\alpha^2v^2g^2 + g(g+1)\alpha^2v^2 + 1 + 2\alpha vg).$

To obtain the expression of the optimum value of w_1 and w_2, partial differentiate of MSE(t_{mi}) with respect to w_1 and w_2 and then equate the result to zero, we get

$$w_1 = \frac{C - w_2E}{A} \quad w_2 = \frac{D - w_1E}{B}$$

The expression of optimum values of $w_i (i = 1,2)$ are obtained as

$$w_1^{opt} = \frac{BC - DE}{AB - E^2} \quad w_2^{opt} = \frac{AD - CE}{AB - E^2}$$

Substituting optimum values of w_1^{opt} and w_2^{opt} in (1.16) we get

$$MSE(t_{mi})_{\min} = \bar{Y}^2\left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \tag{18}$$

Table-2

(Family of t_m for distinct choice of a, b$(\alpha = 1)$

g	a	b	Estimators
1	1	1	$t_{m1} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)\frac{(\bar{\bar{X}}+1)}{(\bar{x}+1)}$

	1	-1	$t_{m2} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)\frac{(\bar{X}-1)}{(\bar{x}-1)}$
	1	0	$t_{m3} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)\frac{(\bar{X})}{(\bar{x})}$
	0	1	$t_{m4} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)$
-1	1	1	$t_{m1} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)\left(\frac{\bar{X}+1}{\bar{x}+1}\right)^{-1}$
	1	-1	$t_{m2} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)\left(\frac{\bar{X}-1}{\bar{x}-1}\right)^{-1}$
	1	0	$t_{m3} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)\left(\frac{\bar{X}}{\bar{x}}\right)^{-1}$
	0	1	$t_{m4} = \bar{y}\left(w_1 + w_2\left(\frac{\bar{X}}{\bar{x}}\right)\right)$

Mathematical efficiency comparison between the proposed estimator and existing estimators

The proposed estimator is compared with the existing estimators were listed below.

1. Comparison between proposed estimator and usual sample mean

Proposed estimator compared with the sample mean $V(\bar{y})$, resulting in a proposed estimator is better(ref:Table 1).

$$MSE(t_{mi}) < V(\bar{y})(i = 1,2,3,4)$$
$$\theta C_Y^2 - \left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \geq 0$$

2. Comparison between proposed estimator and Cochran(T_r)

Proposed estimator compared with the cochran estimator (T_r), resulting in a proposed estimator is better(ref:Table 1).

$$MSE(t_{mi}) < MSE(T_r)(i = 1,2,3,4)$$
$$\theta(C_Y^2 + C_X^2 - 2\rho C_Y C_X) - \left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \geq 0$$

3. Comparison between proposed estimator and Murthy [7](T_P)

Proposed estimator compared with the Murthy estimator (T_P), resulting in a proposed estimator is better(ref:Table 1).

$$\text{MSE}(t_{mi}) < \text{MSE}(T_P)(i = 1,2,3,4)$$
$$\theta(C_Y^2 + C_X^2 + 2\rho C_Y C_X) - \left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \geq 0$$

4. Comparison between proposed estimator and Bahl and Tujeta (T_{1-rat})

Proposed estimator compared with the Bahl and Tujeta ratio estimator (T_{1-rat}), resulting in a proposed estimator is better(ref:Table 1).

$$\text{MSE}(t_{mi}) < \text{MSE}(T_{1-rat})(i = 1,2,3,4)$$
$$\theta\bar{Y}^2\left[C_y^2 + \frac{C_x^x}{4} - \rho C_y C_x\right) - \left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \geq 0$$

5. Comparison between proposed estimator and Bahl and Tujeta(T_{2-pro})

Proposed estimator compared with the Bahl and Tujeta product estimator $\left(T_{2-pro}\right)$, resulting in a proposed estimator is better(ref:Table 1).

$$MSE(t_{mi}) < MSE\left(T_{1-pro}\right)(i = 1,2,3,4)$$
$$\theta\bar{Y}^2\left[C_y^2 + \frac{C_x^x}{4} + \rho C_y C_x\right) - \left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \geq 0$$

6. Comparison between proposed estimator and Yunusa et al.,(T_M)

Proposed estimator compared with the Yunisa et al estimator (T_M), resulting in a proposed estimator is better(ref:Table 1).

$$MSE(t_{mi}) < MSE(T_M)(i = 1,2,3,4)$$
$$\theta\bar{Y}^2 C_y^2(1 - \rho) - \left(1 - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) \geq 0$$

Empirical Study

The two numerical data used for the numerical study ([1] were
Population 1:
Source:(Cochran 1977)
The auxiliary variable X is the number of rooms and the study variable Y is the number of persons.

N = 10	n = 4	$\bar{Y} = 5.920$	$\bar{X} = 3.590$
$C_y = 0.144$	$C_x = 0.128$	$\rho_{yx} = 0.680$	

Population 2:
Source :(Murthy 1967,p.228[7]) The auxiliary variable X is the output of the 80 factories and the study variable is the fixed capital.

N = 80	n = 20	$\bar{Y} = 11.264$	$\bar{X} = 51.826$
$C_y = 0.750$	$C_x = 0.354$	$\rho = 0.941$	

Table-3
(MSE's and PRE's of Various estimators under SRSWOR)

Estimators	MSE - Popn. 1	PRE - Popn.1	MSE - Popn. 2	PRE - Popn. 2
$\bar{y}$	12.6366	100.00	26.7633	100.00
$\overline{y_R}$	1.7874	157.5875	8.9518	298.9715
$\overline{y_p}$	54.4632	33.948	56.4996	47.3689
t_1	1.6172	161.3546	16.3669	163.5205
t_2	52.0376	56.3282	40.1408	66.6734
T_M	1.4399	173.1528	3.0649	873.2175

Percentage relative efficiency(PRE)formula is calculated below using,

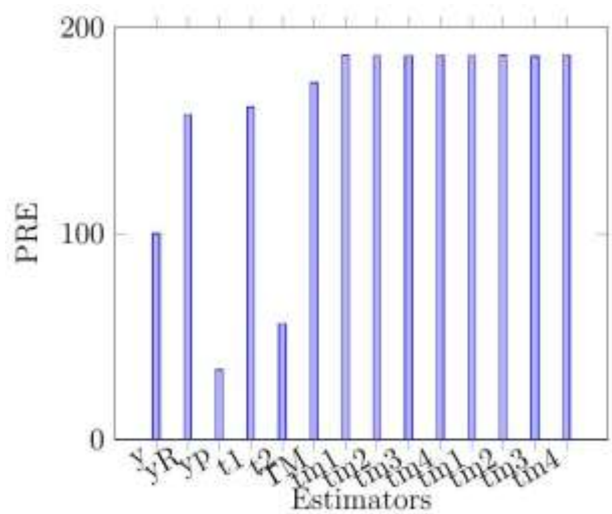
$$\text{PRE}(*, \bar{y}) = \frac{V(\bar{y})}{MSE(*)} * 100$$

Table-4

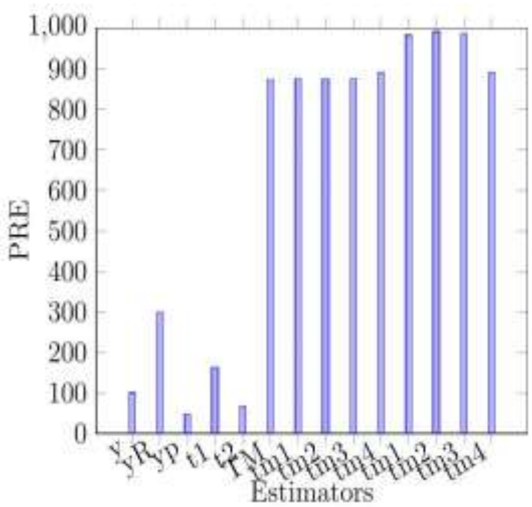
(MSE's and PRE's of Proposed estimators under SRSWOR) Ratio estimator	g = 1			
Estimators	MSE-Popn.1	PRE-Popn.1	MSE-Popn.2	PRE-Popn.2
t_{m1}	0.05851	186.32	0.30581	875.17
t_{m2}	0.05854	186.21	0.30591	874.88
t_{m3}	0.05852	186.28	0.30586	875.02

t_{m4}	0.05845	186.51	0.30033	891.12
Product estimator	$g = -1$			
Estimators	MSE-Popn.1	PRE-Popn.1	MSE-Popn.2	PRE-Popn.2
t_{m1}	0.05856	186.14	0.27221	983.18
t_{m2}	0.05849	186.38	0.26977	992.07
t_{m3}	0.0586	186.02	0.27103	987.45
t_{m4}	0.05845	186.51	0.30033	891.12

Graph-1:PRE of modified estimators for popn 1



Graph-2:PRE of modified estimators for popn 1,popn 2



From Graph 1 and Graph 2, the PRE values for population 1 and population 2 were higher for the proposed estimator in Table:4 than the existing estimators given in Table 3.

Conclusion

The family of estimators among proposed families $t_{mi}(\mathrm{i} = 1,2,3,4)$ for the distinct value of $\mathrm{a, b, g}$ is more efficient than $\bar{y}, \overline{y_R}, \overline{y_p}$ and t_1, t_2, T_M. And it is uniformly efficient. Hence, to obtain the greater efficiency, the proposed estimator is suggested.

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